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ANALYTICAL STUDY OF THERMAL STRESSES IN A SOLID CIRCULAR CYLINDER

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ABSTRACT

this paper is concerned with the study of the thermoelastic problem of a solid circular cylinder occupying the space; $D : 0 \le r \le b,\, 0 \le z \le h$, with radiation-type boundary conditions. We apply integral transform techniques to find the temperature distribution , thermoelastic displacement and thermal stresses . Any particular cases of special interest can be derived by assigning suitable values to the parameter and functions in the expressions.

Key words : thermoelastic problem , solid circular cylinder , thermal Stress ,radiation type , integral transform , temperature distribution , thermoelastic displacement and thermal stresses .

1. INTRODUCTION

The direct and inverse problem of thermoelasticity of thin circular plate are considered by **Nowacki. W**. The quasi-static thermal stresses in circular plate subjected to arbitrary initial temperature on the upper face with lower face at zero temperature has determined by **Wankhede**. **Roy Choudhuri** has succeeded in determining the quasi-static thermal stresses in a circular plate subjected to transient temperature along the circumference of circular upper face with lower face at zero temperature and the fixed circular edge thermally insulated. In all aforementioned investigations, they have not considered any thermoelastic problems with boundary conditions of radiation type. This paper is concerned with the transient thermoelastic problem of a solid circular cylinder occupying the space $D : 0 \le r \le b,\, 0 \le z \le h$, with radiation-type boundary conditions.

2. STATEMENT OF THE PROBLEM

Consider solid circular cylinder of length h occupying the space $D : 0 \le r \le b,\, 0 \le z \le h$, the material is homogeneous and isotropic. The differential equation governing the displacement function $U(r,z,t)$ is

$$\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} = (1+v)\ a_t T \qquad (1)$$

Where v and a_t are Poisons ratio and the linear coefficient of thermal expansion of the material of the cylinder and T is the temperature of the solid circular cylinder satisfying the differential equation

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k}\frac{\partial T}{\partial t} \qquad (2)$$

subject to the initial condition

$\Re_t(T,\ 1,\ 0,\ 0)\ =\ 0$ for all $0 \le r \le b,\ 0 \le z \le h$(3)

And the boundary conditions

$$\Re_r(T,\ 1,\ 0,\ b)\ = 0 \text{ for all } 0 \le z \le h, t > 0 \qquad (4)$$

$$\Re_z(T,1,\ k_1,\ 0)\ = \exp[\omega t]\quad \delta(r - r^0)$$

$$\Re_z(T,\ 1,\ k_2,\ h)\ = \frac{Q_0}{\lambda}\exp[\omega t]\,\delta(r - r^0) \quad \text{for all } 0 \le r \le b, t > 0 \qquad (5)$$

The most general expression for these conditions can be given by

$$\Re_v(f,k,\ \bar{\bar{k}},\ \$)\ =\ (\bar{k}f + \bar{\bar{k}}\hat{f})_{v=\$}$$

where the prime (^) denotes differentiation with respect to v, $\delta(r-r^0)$ is the Dirac Delta function having $0 \le r^0 \le b$, $\omega > 0$ is constant, $\frac{Q_0}{\lambda}\exp[\omega t]\,\delta(r-r^0\,)$ is the additional sectional heat available on its surface at $z = 0, h$.

The stress function σ_{rr} and $\sigma_{\theta\theta}$ are given by

$$\sigma_{rr} = -2\mu \frac{1}{r}\frac{\partial U}{\partial r} \tag{6}$$

$$\sigma_{\theta\theta} = -2\mu\ \frac{\partial^2 U}{\partial r^2} \tag{7}$$

where μ is the Lame's constant, while each of the stress functions σ_{rr}, σ_{zz} and $\sigma_{\theta z}$ are zero within the solid circular cylinder in the plane state of stress.

The equations (1) to (7) constitute the mathematical formulation of the problem under consideration.

3. SOLUTION OF THE PROBLEM

By applying the finite Hankel transform to the equations (2) ,(3) ,(4)and (5) and then applying the inversion theorem one obtains the expression for the temperature distribution function as

$$T(r,z,t) = \frac{2}{h}\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{[(-1)^{m+1}+1]Q_0\left(\frac{km\pi}{h}\right)\ r^0\ f_0(\xi_n,\ r^0)}{\left[k\left(\xi_n^2+\frac{m^2\pi^2}{h^2}\right)-\omega\right]\lambda}\ \Big[\exp[\omega t] - \exp\left[-k\left(\xi_n^2+\frac{m^2\pi^2}{h^2}\right)t\right]\Big]\sin\left(\frac{m\pi z}{h}\right)\ f_0(\xi_n, r) \tag{8}$$

4. DETERMINATION OF THERMOELASTIC DISPLACEMENT

Substituting value of $T(r,\ z,t)$ from equation (8) in equation (1), one obtains the thermoelastic displacement function $U(r,z,t)$ as

$$U(r,z,t) = -(1+v)\ a_t\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{[(-1)^{m+1}+1]\ Q_0\left(\frac{km\pi}{h}\right)\ r^0\ f_0(\xi_n, r^0)}{\left[k\left(\xi_n^2+\frac{m^2\pi^2}{h^2}\right)-\omega\right]\xi_n^2\lambda}\ \Big[\exp[\omega t] - \exp\left[-k\left(\xi_n^2+\frac{m^2\pi^2}{h^2}\right)t\right]\Big]\sin\left(\frac{m\pi z}{h}\right)\ f_0(\xi_n, r) \tag{9}$$

5. DETERMINATION OF THERMAL STRESSES

Substituting value of $U(r,\ z,t)$ from equation (9) in equation (6) and (7) one obtains thermal stresses as

$$\sigma_{rr} = \frac{2}{h}(1+v)\ a_t\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{[(-1)^{m+1}+1]Q_0\left(\frac{km\pi}{h}\right)\ r^0\ f_0(\xi_n, r^0)}{\xi_n^2\left[k\left(\xi_n^2+\frac{m^2\pi^2}{h^2}\right)-\omega\right]\lambda}\ \Big[\exp[\omega t] - \exp\left[-k\left(\xi_n^2+\frac{m^2\pi^2}{h^2}\right)t\right]\Big]\ \sin\left(\frac{m\pi z}{h}\right)\times\frac{2\mu}{r}\left[\frac{\sqrt{2}}{b}\frac{J_1(\xi_n r)}{J_1(\xi_n b)}\right] \tag{10}$$

$$\sigma_{\theta\theta} = \frac{2}{h}(1+v)\ a_t\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{[(-1)^{m+1}+1]Q_0\left(\frac{km\pi}{h}\right)\ r^0\ f_0(\xi_n, r^0)}{\xi_n^2\left[k\left(\xi_n^2+\frac{m^2\pi^2}{h^2}\right)-\omega\right]\lambda}\ \Big[\exp[\omega t] - \exp\left[-k\left(\xi_n^2+\frac{m^2\pi^2}{h^2}\right)t\right]\Big]\ \sin\left(\frac{m\pi z}{h}\right)\times\xi_n\left[\frac{J_0(\xi_n r)}{J_1(\xi_n b)}-\frac{J_1(\xi_n r)}{(\xi_n r)J_1(\xi_n b)}\right] \tag{11}$$

6. SPECIAL CASE AND NUMERICAL RESULTS

Take $k = 86$, $h = 2$cm, $b = 4$ cm, $t = 1\ sec$, $r^0 = 1$ cm $\omega = 1$. Substitute this values in (20), one obtains

$$T(r,z,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \frac{[(-1)^{m+1}+1]Q_0\left(\frac{0.86\times 3.14\times m}{2}\right) f_0(\xi_n, r^0)}{(0.86)\left[\left(\xi_n^2+\frac{m^2(3.14)^2}{2^2}\right)-1\right]}\left[e^1 - e^{-.86\left(\xi_n^2+\frac{m^2(3.14)^2}{2^2}\right)}\right]\times$$

$$sin\left(\frac{m(3.14)z}{2}\right) f_0(\xi_n, r) \qquad (12)$$

7. CONCLUSION

The temperature distribution, displacement and thermal stresses of solid circular cylinder are investigated with known radiation-type boundary conditions and integral transform technique . Any particular cases of special interest have been derived by assigning suitable values to the parameters and functions in the expressions.

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